

Research article

# MATHEMATICAL MODEL TO PREDICT DEGREE OF SATURATION INFLUENCED BY VELOCITY IN CHICOCO CLAY FORMATION AT AHOADA EAST, RIVERS STATE OF NIGERIA

Eluozo, S. N.

Subaka Nigeria Limited, Port Harcourt, Rivers State of Nigeria  
Director and Principal Consultant, Civil and Environmental Engineering,  
Research and Development  
E-mail: [soloeluzo2013@hotmail.com](mailto:soloeluzo2013@hotmail.com)

---

## Abstract

Chicoco clay in the deltaic formation are predominantly found in wet land areas, there are depositions of clay at other deltaic environments, these formations have various plasticity, which are based on the stratification of the formation under the influences of alluvia deposition, including other environmental influences in the deltaic environments. The degree of saturation on Chicoco clayey formation has been expressed to deposit at different rate that should express high degree of plasticity, clay form are not always friendly with several construction activities. There are lots of challenges on the management of clay formation in construction activities, these are based on the rate of the plasticity in the soil, mathematical model were developed to monitor the rate of saturation in Chicoco clay, the study is imperative because the rate of saturation through absorption of fluid can be determined at various depositions, this can also determine the low rate of permeability and velocity of fluid deposition in the study area. **Copyright © WJSWAP, all rights reserved.**

**Keywords:** Mathematical model, degree of saturation, velocity and Chicoco clay

---

## 1. Introduction

A floodplain forest grows in specific hydrological conditions. Its biological particularity, combined with its ecological and economic function, is a trigger mechanism for the study of patterns of its internal processes. Water is the most important factor in these processes. Soil water is usually a very dynamic component. In case of floodplain

forests, this dynamics is less distinct because soil is completely saturated with water for long periods. This feature is directly reflected on the reduced soil air dynamics [3]. The annual and seasonal water component dynamics in the soil of a floodplain forest are particularly interesting in a highly indented microrelief. Apart from the microrelief, this variability also depends on soil properties and on the characteristics of the forest plant cover: tree species, age and distribution in space [4, 5, 2, and 8]. Soil moisture and soil clay content have a vital effect on the organic carbon content and microbiological mass, as well as on the degree of nitrification [4,9]. Wessolek (2001) deals with the problem of determining (measuring) the water component in a forest ecosystem, while [7,12] gained very valuable insights into terrestrial conditions. This problem is also manifested in modelling ecosystem water balance [10]. In the last two decades the development of a water balance model in a forest ecosystem has received strong impetus by the application of recent methods of monitoring water in the soil [1, 8 and 11]. These methods are based mainly on measuring dielectric soil coefficient [6] by using TDR (Time Domain Reflectometry) or FDR (Frequency Domain Reflectometry) systems. The forest of narrow-leaved ash (*Fraxinus angustifolia*) has a prominent place within the issue of floodplain forest hydrology. In relation to water, narrow-leaved ash is a hygrophytic species. It grows on a range of sites starting from bogged sites, where it forms a boggy forest boundary towards a swamp, to fresh flatland microelevations. Since the floristic-vegetational and forest-economic aspect of an ash forest in different relief positions changes fundamentally, there is a question of the existence of possible differences and of their nature in the soil of the sites with such micro relief. Our research focuses on the dynamics of the degree of soil water saturation in ash stands of a floodplain lowland forest in the growing season. Research is aimed at analyzing the dynamics of the soil water saturation degree during three characteristic evolution phases of ash stands. Data for each evolution phase are dealt with separately and differences among them are analysed [3].

## **2. Theoretical background**

The study of Chicoco clayey material in deltaic environment has not been given serious attention in the study area, the degree of saturation in such formation has lots to observed in several engineering application, the behaviour of clayey formation has lots of challenges in construction applications, high plasticity are normally found in clayey formation, there are several types of clayed, it also reflect on the permeability porosity and void depositions in the soil and water environments. Wet land in most deltaic locations normally deposit Chicoco clayey formation, there degree of saturation are determined by the rate of these stated formation characteristics, porosity, void ratio and permeability, clay normally deposit low degree of these three parameters compared to their soil formations, just as they developed the highest plasticity from any other soil formation. In many instances facts of at least one supplementary parameter, the mathematical distribution of pore sizes in the mass, may assist in interpreting the behaviour of the material. Changes in the pore-size distribution arising from mechanical loading, remolding, chemical treatment, plant root growth, freezing and thawing, or other influences often need to be taken into account in interpreting the formation in terms of experimental results. There are two major approaches currently available for the determination of pore size distributions in porous solids, viz.: forced intrusion of a non-wetting fluid (usually mercury) under pressure, and the "capillary condensation method," which involves interpretation of adsorption or

desorption isotherms to calculate that portion of the sorption that is due to capillary condensation. The latter method is much more difficult experimentally and suffers from the limitation that the maximum pore diameter can be tallied is of the order of 1000 fit, much smaller than many of the pores present in clays. In consequence, no further discussion is made of the capillary condensation approach and the remainder of this paper is concerned with mercury intrusion measurements. For most porous substances, the critical requirement that the pores be empty at the start normally is met by oven drying the sample, followed by evacuation to comparatively low pressures. While the effects of oven drying and evacuation are of little consequence for most materials, partly or completely saturated clay (or soil) bodies may shrink appreciably on drying, decreasing the total porosity by an often significant amount. Details of the shrinkage process depend on the initial moisture content and degree of saturation, the microstructure of the clay, and probably the details of the drying process itself [3].

### 3. Governing Equation

$$Vt \frac{\partial s}{\partial L} = \frac{\partial s}{\partial t} \frac{V_w}{V} + Kt \frac{\partial s}{\partial L} \dots\dots\dots (1a)$$

Nomenclature

- Vt = Velocity of flow [LT<sup>-1</sup>]
- $S = \frac{V_w}{V}$  = Degree of saturation [-]
- Kt = Permeability [LT<sup>-1</sup>]
- L = Depth
- T = Time

The expression here is the developed governing equation which were generated from the system that determine the deposition of degree of saturation in soil and water environment, the expression denoted with mathematical symbols were to be derived considering the behaviour of Chicoco clayey formation on the attaining degree of saturation in different conditions. The governing equation expresses the parameters that may definitely deposit relationship with degree of saturation in the system.

Simplifying the expression, let  $\frac{V_w}{V}$  denote as  $\tau$  so that the equation can be written as:

$$Vt \frac{\partial s}{\partial L} = \tau \frac{\partial s}{\partial L} + Kt \frac{\partial s}{\partial L} \dots\dots\dots (1b)$$

$$Vt \frac{\partial s}{\partial L} - \frac{\partial s}{\partial t} = \frac{\partial s}{\partial L} + \tau + Kt \dots\dots\dots (2)$$

$$(Vt - 1) \frac{\partial s}{\partial L} = \frac{\partial s}{\partial L} + \tau + K \dots\dots\dots (3)$$

$$(Vt - 1) \frac{\partial s}{\partial L} = \frac{\partial s}{\partial t} \dots\dots\dots (4)$$

$$0 = \frac{\partial s}{\partial L} + \tau + K \dots\dots\dots (5)$$

$$\text{i.e. } \frac{\partial s}{\partial t} = -\tau - Kt \dots\dots\dots (6)$$

From (5) integrate directly, we have

$$S = (-\tau - Kt)t + S_1 \dots\dots\dots (7)$$

From (6)

$$\frac{\partial s}{\partial L} = \frac{\partial s}{\partial t}$$

$$\text{Let } S = LT \dots\dots\dots (8)$$

$$\frac{\partial s}{\partial L} = Z^1 T \dots\dots\dots (9)$$

$$\frac{\partial s}{\partial L} = ZT^1 \dots\dots\dots (10)$$

Substitute (9) and (10) into (3), we have

$$(Vt-1)Z^1 T = (\tau - Kt)ZT^1 \dots\dots\dots (11)$$

$$(Vt-1)\frac{Z^1}{Z} = (\tau - Kt)\frac{T^1}{T} = \phi \dots\dots\dots (12)$$

$$(Vt-1)\frac{Z^1}{Z} = \phi \dots\dots\dots (13)$$

$$(\tau - Kt)\frac{T^1}{T} = \phi \dots\dots\dots (14)$$

$$\text{From (13)} \frac{Z^1}{Z} = \frac{\phi}{(Vt-1)} z \dots\dots\dots (15)$$

$$\text{Ln } Z = \frac{\phi}{(Vt-1)} z + S_2 \dots\dots\dots (16)$$

$$Z = A e^{\frac{\phi}{(Vt-1)} z} \dots\dots\dots (17)$$

The derived solution in (17) is the expressed model that is established to monitor the degree of saturation under the influences of velocity and porosity, though the deposition of these two parameters may be very low, but the rate of saturation may also pressure them in most conditions to have an influence in any amount of percentage deposited in Chicoco clayey formation. These expressed model are also determined at different depths of the predominant Chicoco clayey depositions in the deltaic formation. Therefore, the influences of these two parameters were considered in the system to generate the derived solution in equation (17).

From (14)

$$(\tau + Kt) \frac{T}{T} = \phi$$

$$T = \frac{\phi}{(\tau + Kt)} \dots\dots\dots (19)$$

$$\ln T = \frac{\phi}{(\tau + Kt)} t + S_3 \dots\dots\dots (19)$$

$$T = B \ell^{\frac{\phi}{\tau + Kt} t} \dots\dots\dots (20)$$

Put (17) and (20) into (8), yield

$$S_2 = A \ell^{\frac{\phi}{(Vt-1)} z} \bullet B \ell^{\frac{\phi}{\tau + Kt} t} \dots\dots\dots (21)$$

$$S_2 = AB \ell^{\left( \frac{z}{(Vt-1)} + \frac{t}{\tau + Kt} \right) \phi} \dots\dots\dots (22)$$

Hence general solution becomes:

$$S [LT] = S_1 + S_2 \quad S [LT] = AB \ell^{\left( \frac{z}{(Vt-1)} + \frac{t}{\tau + Kt} \right) \phi}$$

The behaviour of Chicoco clayey formation under the deposition of degree of saturation has been expressed in the derived solution, the development of these model were to monitor the rate of saturation at different formations, which has established various degree of saturation under the influences of deltaic formation reflecting the degree of void ratio, permeability and porosity. These parameters experienced little pressure of their various reactions in predominant depositions of Chicoco clay in the study location. The deposition of these formations are also

influenced by the deltaic nature of the environment, these were integrated on the derived solution that generated the final model for the study.

#### 4. Conclusion

The developed model was to monitor the degree of saturation at various formations in the deltaic environment of Ahoada East. the structure of soil has expressed its predominant parameters that definitely pressure the increasing degree of saturation, the expressed model generated were able to note the deposited variables in the system.

The degree of saturation,  $S$ , has an important influence on the soil behaviour. It is defined as the ratio of the volume of water to the volume of voids. The distribution of the volume phases may be expressed in terms of  $e$  and  $S$ , and by knowing the unit weight of water and the specific gravity of the particles thus the distributions by weight may also be determined as indicated in. When all the voids are filled with water the bulk unit weight is identical to the saturated unit weight,  $\gamma_{sat}$ , and when all the voids are filled with air the bulk unit weight is identical with the dry unit weight,  $\gamma_{dry}$ . Note that in discussing soils that are saturated it is common to discuss their dry unit weight. This is done because the dry unit weight is simply related to the voids ratio; it is a way of describing the amount of voids. The moisture content,  $m$ , is a very useful quantity because it is simple to measure. It is defined as the ratio of the weight of water to the weight of solid material.

#### References

- [1] Bell, J.P., Dean, T.J., Hodnett, M.G., 1987: Soil moisture measurement by an improved capacitance technique. Part II. Field techniques, evaluation and calibration. *J. Hydrol.*, 93: 79–90.
- [2] Bruckner, A., Kandeler, E., Kampichler, C., 1999: Plot-scale spatial pattern of soil water content, pH, substrate induced respiration and N mineralisation in a temperate coniferous forest. *Geoderma*, 93: 207–223.
- [3] Nikola P, Igor A, Anamarija J, Darko B. 2007 :The degree of soil water saturation in the narrow-leaved ash (*fraxinus angustifolia* va h l.) floodplain forest *Ekologia (Bratislava)* Vol. 26, No. 3, p. 258–272.
- [4] Hardtle, W., von Oheimb, G., Westphal, C., 2003: The effects of light and soil conditions on the species richness of the ground vegetation of deciduous forests in northern Germany (Schleswig-Holstein). *For. Ecol. Manag.*, 182: 327–338.
- [5] Hewlett, J.D., 1982: Principles of forest hydrology. The University of Georgia Press, Athens, GA, 183 pp.
- [6] Hilhorst, M.A., 1997: Dielectric characterisation of soil. Doctoral thesis. Wageningen Agricultural University, Wageningen, 136 pp.
- [7] Hormann, G., Meesenburg, H., 2000: Die Erfassung und Modellierung des Wasserhaushaltes im Rahmen des Level-II-Programms in Bundesrepublik Deutschland. *Forstarchiv, Hannover*, 71: 70–75.
- [8] Dean, T.J., Bell, J.P., Baty, A.J.B., 1987: Soil moisture measurement by an improved capacitance technique. Part 1. Sensor design and performance. *J. Hydrol.*, 93: 67–78.
- [9] Eaton, W.D., 2000: Microbial and nutrient activity in soils from three different subtropical forest habitat in Belize, Central America before and during the transition from dry to wet season. *Appl. Soil Ecol.*, 16: 219–227.

- [10] Evans, S.P., Thomas, R.M., Hollis, J.M., Brown, C.D., 1999: SWBCM: a soil water balance capacity model for environmental applications in the UK. *Ecol. Modell.*, 121: 17–49.
- [11] Schume, H., Jost, G., Katzensteiner, K., 2003: Spatio-temporal analysis of the soil water content in a mixed Norway spruce (*Picea abies* (L.) Karst.) – European beech (*Fagus sylvatica* L.) stand. *Geoderma*, 112: 273–287.
- [12] Strohbach, B., Degenhardt, A., 1999: Bodenhydrologische Untersuchungen auf Level-II-Standorten in Brandenburg. *Beitrage F. Forstwirtschaft und Landschaftsokologie*, Berlin, 33: 114–117.
- [13] Seyfried, M.S., Murdock, M.D., 2001: Response of a new soil water sensor to variable soil, water content, and temperature. *Soil Sci. Soc. Am. J.*, 65: 28–34.
- [14] Likens, G.E., Bormann, F.H., Pierce, R.S., Eaton, J.S., Johnson, N.M., 1995: Biogeochemistry of a forested ecosystem. Springer, New York, 146 pp.
- [15] Mallants, D., Mohanty, B.P., Jaques, D., Feyen, J., 1996: Spatial variability of hydraulic properties in a multilayered soil profile. *Soil Sci.*, 161: 167–181.
- [15] Wessolek, G., 2001: Bestimmung von Wasserhaushaltskomponenten in Waldokosystemen. Probleme und Erfahrungen. *Beitr. Forstwirtsch. Landschaftsokol.*, 35: 2–4.
- [16] Sidney D pore size distributions in clays *Clays and Clay Minerals*, 1970, Vol. 18, pp. 7-23.